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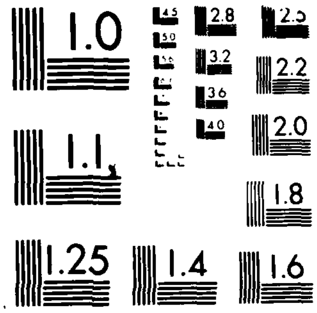

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AN INTEGRATED, OPTIMIZATION-BASED APPROACH TO THE DESIGN AND CONTROL OF LARGE SPACE STRUCTURES

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Principle Investigators

October 1, 1983 to May 1, 1984

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The investigators proposed to consider the design of large space structures which are required to perform large amplitude maneuvers at the end of which they are required to remain "locked" on a target. They proposed to deal with the pointing of the LSS in two stages. In the first stage, the control task is to rapidly redirect the pointing direction of the LSS reference axis, e.g., the line-of-sight of a telescope or antenna, to a desired target direction. At the end of the first stage, which is to be carried out by open loop optimal control, the large motions of the LSS induced by the maneuver must "quiet down" so that the control of the LSS can be transferred to a linear, closed loop control system. The task of the latter is to damp out the induced structural vibrations, to suppress disturbance effects (e.g., caused by running cooling water through pipes), and, finally, to lock the pointing direction on the target. For the purpose of obtaining a tractable model problem for the research, the investigators shall initially assume that
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ITEM #19, ABSTRACT, CONTINUED: the LSS is a beam. The investigators shall use a nonlinear beam model for the large motions, and they shall use a linear beam model to describe the small displacements as a perturbation around the equilibrium rigid body configuration. In fully developing the model problem, described within, the investigators shall first develop the equations of motion for a beam under large and small displacement conditions. They shall then use the resulting equations in transcribing sample design specifications into infinite systems of inequalities.

An Integrated, Optimization-Based Approach

to the Design and Control of Large Space Structures.

Elijah Polak, Karl S. Pister, Robert L. Taylor

PROGRESS REPORT for the Period October 1, 1983 to May 1, 1984.

The following activities took place in the report period.

1. In January 1984, we offered a three day workshop on the fundamentals of semi-infinite optimization in engineering design. The workshop was attended by 20 participants from Air Force Navy and NASA laboratories, some of their contractors and a few civilians from industry. A memorandum [1] 1was prepared for distribution to the participants. This memorandum presents a new theory dealing with the construction of semi-infinite optimization algorithms for engineering design. A sequel to this memorandum is is currently being prepared. It is expected that the two memoranda will serve as a basis for an invited review paper to be published by SIAM reviews.

2. We have examined various alternatives in the formulation of the optimal integrated design of a large space structure and its control system. We have settled on a model problem which is to serve as a basis for our research. This model problem contains a number of features that have previously not been dealt with in the context of optimization-based design and it has led to a number of very challenging and exciting new problems. Our model problem is presented in Appendix 1.

3. Three papers have been completed which deal with various aspects of optimization-based control system design.

In [2] we describe our recent work on developing a coherent methodology for

optimization based control system design as well as the progress that has been made on our control system design software package, DELIGHT.MIMO.

In [3] we present a survey of the basic aspects of our methodology of design of linear multi-variable control systems via semi-infinite optimization. Specific topics treated are (i) data-base and simulation requirements, (ii) techniques for the transcription of design specifications into semi-infinite inequalities, and (iii) semi-infinite optimization algorithms for control system design.

In [4] We show that worst case design of control systems with both parametric and unstructured uncertainty leads to a new class of semi- infinite optimization problems. We present a three phase semi-infinite optimization algorithm based on the theory developed in [1] for solving this class of problems.

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[1] E. Polak, "Notes on the Mathematical Foundations of Nondifferentiable Optimization in Engineering Design", University of California, Berkeley, Electronics Research Laboratory Memo No. UCB/ERL M84/15, 2 February 1984.

[2] E. Polak and D. Q. Mayne, "Theoretical and Software aspects of Optimization-Based Control System Design", University of California, Berkeley, Electronics Research Laboratory Memo No. UCB/ERL M84/23, 1 March 1984. to appear in Proc. Sixth International Conference on the Analysis and Optimization of Systems, Nice, France, June 19-22, 1984.

[3] E. Polak, D. Q. Mayne and D. M. Stimler, "Control System Design via Semi-Infinite Optimization: A Review", University of California, Berkeley, Electronics Research Laboratory Memo No. UCB/ERL M84/35, 3 April 1984. to appear in a Special Issue of Proceedings of IEEE

[4] E. Polak and D. M. Stimler, On the Design of Linear Control Systems with Plant Uncertainty via nondifferentiable Optimization", to appear in Proc. IFAC Congress, Budapest, Hungary, July 1984.

APPENDIX 1: MODEL PROBLEM OF AN INTEGRATED LSS OPTIMAL DESIGN.

(i) Introduction.

As we have already stated in the introduction, we propose to consider the design of large space structures which are required to perform large amplitude maneuvers at the end of which they are required to remain "locked" on a target. We propose to deal with the the pointing of the LSS in two stages. In the first stage, the control task is to rapidly redirect the *pointing direction* of the LSS reference axis, e.g,. the line-of-sight of a telescope or antenna, to a desired target direction. At the end of the first stage, which is to be carried out by open loop optimal control, the large motions of the LSS induced by the maneuver must "quiet down" so that the control of the LSS can be transferred to a linear, closed loop control system. The task of the latter is to damp out the induced structural vibrations, to suppress disturbance effects (e.g., caused by running cooling water through pipes), and, finally, to lock the pointing direction on the target.

For the purpose of obtaining a tractable model problem for our research, we shall initially assume that the LSS is a beam. We shall use a nonlinear beam model for the large motions, and we shall use a linear beam model to describe the small displacements as a perturbation around the equilibrium rigid body configuration.

In fully developing our model problem, below, we shall first develop the equations of motion for a beam under large and small displacement conditions. We shall then use the resulting equations in transcribing sample design specifications into infinite systems of inequalities.

(ii) General Considerations in Modeling.

The simulation of a large space structure, under conditions outlined above, involves the analysis of a system which is subjected to large motions about the center of mass and to large deformations. In order to model the LSS response during manoeuvers

and to couple effectively with appropriate control and design algorithms, the structural model must be chosen carefully. There are a number of aspects to this problem which deserve special consideration and research. For space structures which have large motions with respect to the center of mass, the "rigid body" equations for angular momentum will contain time varying "inertia" parameters. Accordingly, an analysis procedure which simulates the response in terms of the "rigid body" response with superposed large motions may not be the most efficient approach. Furthermore, even in the simplified situation when a small motion is superposed on large rigid body motions, significant terms may be omitted if the theory is not properly deduced from a linearization of the more general non-linear forms.

For the purpose of getting preliminary research results, we will use a large displacement three dimensional beam model which includes the salient features for modeling the response of LSS. A theory was developed by Kirchhoff and described, for the static case, in the treatise on elasticity by Love [Lov.1]. Other significant contributions to the theory of beams are contained in the references [Ant.1, Gre.1, Nag.2, and Sim.1]. In the discussion which follows we include the appropriate inertia terms in the theory.

(iii) Equations of Motion of a Beam: Large Displacements.

A typical LSS manoeuver is shown in Figure 1 where a structure which is initially assumed to be in a "quiet" state is subjected to control forces which are to produce a final state which is also "quiet".
During the intermediate times of the manoeuver, the motions in the structure may produce large relative displacements, as shown in Figure 1 for time t'.

For the simplified beam model of a space structure this motion may be described by the relation (e.g., see Figure 2)

$$\mathbf{y}(t,\mathbf{z}) = \varphi_0(t,z_1) + \sum_{i=2}^{3} z_i \tau_i(t,z_1) \tag{2.1}$$

where $\mathbf{y}$ describes the deformed position of the beam, in terms of the vector $\mathbf{z} = (z_1, z_2, z_3)$,, where $z_1 \in [0, l]$ is a parameter defining position along the axis of the beam, z_2, z_3 are coordinate positions in the cross section A, (i.e., the vector $(z_2, z_3) \in A \subset \mathbb{R}^2$), along the directions defined by the unit vectors $\tau_2(t,z_1)$ and $\tau_3(t,z_1)$, t is time, and $\varphi_0(t,z_1)$ is the deformed position of the beam axis. We assume, for simplicity, that the tangent to the deformed axis of the beam satisfies the first Frenet formula

$$\tau_1(t,z_1) = \frac{\partial}{\partial z_1}\varphi_0(t,z_1) \tag{2.2}$$

where $\frac{\partial}{\partial z_1}$ is the derivative along the beam axis, and the unit vectors τ_i in the deformed beam are mutually orthogonal (i.e., plane cross sections remain normal to the beam axis during all deformations). Accordingly, we obtain the kinematic theory of beams due to Kirchhoff which is described in Love's classical treatise [Lov.1] and summarized below (for an expanded description including effects of shear deformation see [Sim.2]). The motion for this theory is completely characterized by φ_0 and τ_i ,i=1,2,3, as shown in Figure 3.

The balance of linear momentum for this theory is given by the relation

$$\frac{\partial}{\partial z_1}\mathbf{f}(t,z_1) + \mathbf{q}(t,z_1) + \mathbf{u}_1(t,z_1) = \rho A(z_1)\frac{d}{dt}\mathbf{v}(t,z_1) \tag{2.3}$$

and the angular momentum equation by

$$\frac{\partial}{\partial z_1}\mathbf{m}(t,z_1) + \frac{\partial}{\partial z_1}\varphi_0(t,z_1)\times\mathbf{f}(t,z_1) + \mathbf{u}_2(t,z_1) = \rho\,\mathbf{I}(z_1)\frac{d}{dt}\omega(t,z_1)\,. \tag{2.4}$$

where $\mathbf{f}(t,z_1)$ and $\mathbf{m}(t,z_1)$ are the stress resultants and stress couples (as shown in Figure 4), $\mathbf{q}(t,z_1)$ is a specified loading (e.g., gravity), $\mathbf{u}_1(t,z_1)$ and $\mathbf{u}_2(t,z_1)$ are the control forces (e.g., see Figure 5 for types of $\mathbf{u}_1$ and $\mathbf{u}_2$

quantities), ρ is the mass density of the beam material, $A(z_1)$ is the cross sectional area, $\mathbf{I}(z_1)$ is the matrix of cross section moment of inertia, $\mathbf{v}(t,z_1)$ and $\omega(t,z_1)$ are the translational velocity and the angular velocity of the local triad τ_i cross section, respectively, and $\frac{d}{dt}$ denotes material time differentiation. In the above, the accelerations are defined in terms of the rate of change of velocity (which are the rates of change of the motion) through

$$\mathbf{v}(t,z_1) = \frac{d}{dt}\varphi_0(t,z_1) \tag{2.5}$$

and

$$\frac{d}{dt}\tau_i(t,z_1) = \omega \times \tau_i(t,z_1) \tag{2.6}$$

It should be noted that this relationship may be parameterized through the use of Euler angles or, to avoid numerical singularities, by using quaternions (see discussion on rigid body motion in section on linear beam models).

As noted previously the kinematic response of the beam is described by φ_0 and the τ_i unit vectors. The τ_i may be defined in terms of the direction cosines with respect to an inertial coordinate frame described by the fixed unit vectors $\mathbf{e}_i$, $i=1,2,3$ of the inertial frame of reference. Accordingly,

$$\begin{bmatrix} \tau_1(t,z_1) \\ \tau_2(t,z_1) \\ \tau_3(t,z_1) \end{bmatrix} = \Lambda(t,z_1)\begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{bmatrix} \tag{2.7}$$

where $\Lambda(t,z_1)$ is the matrix of direction cosines. The rate of twist κ_1 and the changes in curvature κ_2 and κ_3 of the beam, which describe the deformations, may be deduced by differentiating the τ_i's along the beam axis. Thus,

$$\frac{\partial}{\partial z_1}\tau_i(t,z_1) = \left[\frac{\partial}{\partial z_1}\Lambda(t,z_1)\right]\Lambda^T(t,z_1)\,\tau_i(t,z_1) \tag{2.8}$$

where the the twist and changes in curvature are obtained from

$$\boldsymbol{\kappa}(t,z_1) = \left[\frac{\partial}{\partial z_1}\Lambda(t,z_1)\right]\Lambda^T(t,z_1) = \begin{bmatrix} 0 & \kappa_3 & -\kappa_2 \\ -\kappa_3 & 0 & \kappa_1 \\ \kappa_2 & -\kappa_1 & 0 \end{bmatrix} \tag{2.9}$$

The similarity between rigid body mechanics and the beam theory considered here is very evident. Indeed, the beam axis parameter z_1 and time t lead to a direct analogy between quantities relating rates of change along z_1 and time rates of change for the unit vectors τ_i. Accordingly methods which accurately solve for the angular velocity in the rigid y motion problem may also be useful in solving for the beam rotations.

For the kinematics defined by (2.1), the strains $\gamma(t,z_1)$ ar d ied by the relation

$$\gamma(t,z_1) \triangleq \Lambda^T(t,z_1)\frac{\partial}{\partial z_1}\varphi_0(t,z_1) - \tau_1(t,z_1) \tag{2.10}$$

where, due to (2.2), the only nonzero component will be the axial strain.

Appropriate constitutive equations to interrelate the beam stress resultants and stress couples with the strains and deformations may be expressed by

$$\mathbf{f} = \hat{\mathbf{f}}(\gamma,\boldsymbol{\kappa},z_1,t) \tag{2.11a}$$

and

$$\mathbf{m} = \hat{\mathbf{m}}(\gamma,\boldsymbol{\kappa},z_1,t) \tag{2.11b}$$

where $\hat{\mathbf{f}}$ and $\hat{\mathbf{m}}$ are functions of the arguments for elastic materials or, alternatively, they are functionals of the arguments for inelastic (e.g., viscoelastic) materials which can characterize dissipation of energy (i.e., damping). As a special case one may assume linear elastic relations between the stress resultant and beam axis strain

$$\mathbf{f} = \mathbf{B}\gamma, \tag{2.12a}$$

and similarly for the stress-couples and changes in curvatures and twist

$$\mathbf{m} = \mathbf{D}\kappa \tag{2.12b}$$

where **B** and **D** are matrices for the beam which are constant in time..

The response of the beam may be determined by solving the differential equations defined above together with appropriate boundary and initial conditions. The initial conditions for a typical manoeuver may be specified by defining the state vector $\mathbf{x}(t, z_1)$ at an initial time t^0. The state vector is given by the position quantities $\varphi_0(t, z_1)$ and $\tau_i(t, z_1)$, $i=1,2,3$ and the rates of change of the position $\partial\varphi_0(t, z_1)/\partial t$ and of the unit vectors $\partial\tau_i(t, z_1)/\partial t$; that is

$$\mathbf{x} \triangleq (\varphi_0, \tau_1, \tau_2, \tau_3, \frac{\partial\varphi_0}{\partial t}, \frac{\partial\tau_1}{\partial t}, \frac{\partial\tau_2}{\partial t}, \frac{\partial\tau_3}{\partial t}) \tag{2.13}$$

The boundary conditions for a typical LSS simulation will consist of equations which specify the stress resultants and stress couples at the ends of the beam. The boundary conditions may define, for example, beams with attached rigid end masses (we refer to this model as a "dumb bell" structure). The appropriate boundary conditions for this case specify end loads for the beam from the inertial behavior of each rigid end mass.

In general, the response of the beam, at any time, may be specified in terms of the kinematical quantities φ_0 and τ_i by substituting the kinematical measures into the constitutive equations, and then substituting the resulting constitutive equations into the momentum balance equations. If we select appropriate, *finite dimensional* design parameters, $\mathbf{p}_b \in \mathbb{R}^{n_b}$, for the beam (e.g., cross sectional area, A, and moment of inertias, I, etc.) and $\mathbf{p}_a \in \mathbb{R}^{n_a}$ for the actuator locations, we obtain a first order differential equation of the form

$$\dot{\mathbf{x}}(t, z_1) = \mathbf{h}(\mathbf{x}(t, z_1), \mathbf{u}_1(t, z_1), \mathbf{u}_2(t, z_1), \mathbf{p}_b, \mathbf{p}_a) , \tag{2.14}$$

where $\mathbf{h}$ is a differential operator in z_1. The control quantities $\mathbf{u}_1(t, z_1)$ and $\mathbf{u}_2(t, z_1)$ are to be used in positioning the space structure into a particular attitude.

(iv) Equations of Motion of a Beam: Small Displacements.

In the second phase of the manoeuver the structural response may be modeled by linearizing the motion of the LSS about a rigid body response. Accordingly, the beam model can be deduced by describing the small displacements as a linearization of the large displacements relative to a single point, the center of mass, in the beam which describes the rigid body equilibrium configuration. In the following description we let $\mathbf{a}_i(t)$, $i = 1,2,3$, denote the unit vectors of the moving frame attached to the center of mass of the beam structure (i.e., $\mathbf{a}_i(t) = \tau_i(t, z_{1,ref})$ i.e., $z_{1,ref}$ is the position for the center of mass) and, as before, let $\mathbf{e}_i$, $i = 1,2,3$, be the unit vectors of the inertial frame of reference. In addition the beam axis tangent vector, $\mathbf{a}_1(t)$, defines the reference axis which is required to point at the target.

The solution of this phase of the problem requires two parts: (a) the computation of the rigid body response (which may still involve large position and angle changes if the tracking times are of long duration); and (b) the computation of the beam response about the rigid body equilibrium state.

The rigid body motion of a beam about its center of mass is described with respect to the moving coordinate system $\mathbf{a}_i(t)$, by two equations. The translational equation of motion is

$$\dot{\mathbf{v}}(t) = \frac{\mathbf{q}_1(t) + \mathbf{s}_1(\mathbf{u}_1(t), \mathbf{p}_0)}{M} - \omega(t) \times \mathbf{v}(t), \tag{2.15a}$$

and the rotational equation of motion :

$$\dot{\omega}(t) = \mathbf{J}^{-1}\left\{\mathbf{q}_2(\cdot) + \mathbf{s}_2(\mathbf{u}_2(t),\mathbf{p}_a) - \omega\times(\mathbf{J}\omega)\right\} \tag{2.15b}$$

where M is the total mass of the beam, $\mathbf{J}$ denotes the inertia matrix, $\mathbf{q}_1$, $\mathbf{u}_1$ and $\mathbf{q}_2$, $\mathbf{u}_2$ are applied and control forces and couples, respectively, $\mathbf{s}_1(\mathbf{u}_1(t),\mathbf{p}_a)$ and $\mathbf{s}_2(\mathbf{u}_1(t),\mathbf{u}_2(t),\mathbf{p}_a)$ denote the resultant control force and couple on the rigid body, respectively, ω and $\mathbf{v}$ represent the rigid body angular and linear velocities, respectively.

A rate equation describing the evolution of the moving frame with respect to the fixed inertial frame may be written in the form

$$\dot{\chi} = \frac{1}{2}\Omega\chi, \tag{2.16}$$

where χ denotes a quaternion and Ω is a skew-symmetric matrix whose coefficients are composed of the components of the angular velocity, ω_i as follows

$$\Omega = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \tag{2.17}$$

The use of a quaternion avoids singularities which are often encountered in alternative methods (e.g., use of Euler angles).

As noted above, in the second part of the analysis we consider a linearization of the large displacement beam theory about the rigid body response just described. In this analysis we let $w_i(t,x_1)$, $i = 1,2,3$, denote the linearized beam displacements (with respect to the rigid body motions) along the principal axes $\mathbf{a}_i(t)$. The beam is assumed to undergo both axial and transversal motions which are governed by the following partial differential equations for $x_1 \in [0,l]$:

$$m\,\frac{\partial^2 w_1(t,x_1)}{\partial t^2} + c_1\frac{\partial w_1(t,x_1)}{\partial t} - EA\,\frac{\partial^2 w_1(t,x_1)}{\partial x_1^2} = f_1(t,x_1) \tag{2.18}$$

for the axial response; and, for $i = 2,3$,

$$m \frac{\partial^2 w_i(t,z_1)}{\partial t^2} + c_i \frac{\partial w_i(t,z_1)}{\partial t} + EI_i \frac{\partial^4 w_i(t,z_1)}{\partial z_1^4} = f_i(t,z_1) , \quad (2.19)$$

for the flexural response.

Appropriate boundary conditions for a "dumb bell" beam structure are given by:

$$EA \frac{\partial w_1(t,0)}{\partial z_1} - m_1 \frac{\partial^2 w_1(t,0)}{\partial t^2} = 0 , \quad (2.20a)$$

$$EA \frac{\partial w_1(t,l)}{\partial z_1} + m_2 \frac{\partial^2 w_1(t,l)}{\partial t^2} = 0 . \quad (2.20b)$$

for the axial response, and for the flexural response, with $i = 2,3$,

$$EI_i \frac{\partial^2 w_i(t,0)}{\partial z_1^2} - J_{1,i} \frac{\partial^3 w_i(t,0)}{\partial t^2 \partial z_1} = 0 , \quad (2.21a)$$

$$EI_i \frac{\partial^3 w_i(t,0)}{\partial z_1^3} + m_1 \frac{\partial^2 w_i(t,0)}{\partial t^2} = 0 , \quad (2.21b)$$

$$EI_i \frac{\partial^2 w_i(t,l)}{\partial z_1^2} + J_{2,i} \frac{\partial^3 w_i(t,l)}{\partial t^2 \partial z_1} = 0 , \quad (2.21c)$$

$$EI_i \frac{\partial^3 w_i(t,l)}{\partial z_1^3} - m_2 \frac{\partial^2 w_i(t,l)}{\partial t^2} = 0 , \quad (2.21d)$$

In the above expressions m, m_1, m_2 represent the distributed mass and the two end masses, respectively; $J_{1,i}$, $J_{2,i}$ the moments of inertia of the two end masses; c_i the damping coefficients; and EA, EI_i the beam axial and flexural stiffnesses.

The above set of partial differential equations, (2.18) and (2.19), may be solved using separation of variables. Accordingly, let

$$w_i(t,z_1) = \sum_{k=1}^{\infty} \psi_k(t) \eta_{ik}(z_1) \quad (2.22)$$

where $\eta_{ik}(z_1)$ defines the modal shape of the structure and $\psi_k(t)$ is defined as the modal generalized co-ordinate corresponding to the kth mode. To

define the $\psi_k(t)$, a countably infinite set of uncoupled ordinary differential equations results through the projection of the partial differential equations (2.18) and (2.19) onto the modal basis :

$$\ddot{\psi}_k + 2\xi_k\lambda_k\dot{\psi}_k + \lambda_k^2\psi_k = p_k(t) \quad \text{for} \quad k=1,2,\ldots, \tag{2.23}$$

where ξ_k represents the modal damping ratio, λ_k the frequencies of undamped free vibration, and $p_k(t)$ the modal forces (e.g., see [Clo.1]).

For the models (2.23), linear feedback compensators can be designed using semi-infinite optimization. Equations (2.21a, b, c, d) can be easily transformed into a set of first order linear differential equations for use in the design of compensators for the linear closed loop control system. Accordingly, we may write

$$\dot{\mathbf{x}}(t, z_1) = \mathbf{F}(\mathbf{p}_b)\mathbf{x}(t, z_1) + \mathbf{G}(\mathbf{p}_b, \mathbf{p}_a)\mathbf{u}(t, z_1) \tag{2.24a}$$
$$\mathbf{y}(t, z_1) = \mathbf{H}(\mathbf{p}_b, \mathbf{p}_a)\mathbf{x}(t, z_1) + \mathbf{K}(\mathbf{p}_b, \mathbf{p}_a)\mathbf{u}(t, z_1) \tag{2.24b}$$

where $\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2)$ (for details of some previous applications in beams see [Tay.1]).

(iv) Formulation of Design Constraints: Large Displacements.

We assume that the LSS is to be designed taking k specified, large amplitude changes in attitude (manoeuvers) into account. Although one may consider both fixed and free time manoeuvers, we simplify exposition by restricting ourselves to fixed time manoeuvers only. These manoeuvers can be specified by an initial time $t^o{}_i$, a final time $t^f{}_i$, $i = 1,2,\ldots,k$, and two linear equations in the state vector x at these times, of the form:

$$\mathbf{L}^o\mathbf{x}(t^o{}_i, z_1, \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}_b, \mathbf{p}_a) = \mathbf{b}^o{}_i \quad \text{for} \quad i = 1,2,\ldots,k\,; \tag{2.25a}$$
$$\mathbf{L}^f\mathbf{x}(t^f{}_i, z_1, \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}_b, \mathbf{p}_a) = \mathbf{b}^f{}_i \quad \text{for} \quad i = 1,2,\ldots,k\,. \tag{2.25b}$$

Our ***design variables*** for the large manoeuvers are k pairs of control functions $\mathbf{u}_{1,i}, \mathbf{u}_{2,i}$, $i = 1,2,\ldots,k$, and the structural and actuator

parameters $\mathbf{p}_s$ and $\mathbf{p}_a$.

During the large manoeuvers a number of physical constraints must be taken into account.

(a) Control and Parameter Constraints.

The simplest constraints involve bounds on the structural and actuator design parameters and on the controls, which have the form:

$$\underline{\mathbf{p}}_s \le \mathbf{p}_s \le \bar{\mathbf{p}}_s \tag{2.26a}$$

$$\underline{\mathbf{p}}_a \le \mathbf{p}_a \le \bar{\mathbf{p}}_a \tag{2.26b}$$

$$\|\mathbf{u}_{j,i}(\cdot,z_1)\|_\infty \le \bar{u}_j(z_1), \; j = 1,2; \; z_1 \in f_a(\mathbf{p}_a); \; i = 1,2,...,k , \tag{2.26c}$$

where the dependence of location of the control forces is indicated by the use of the set valued function f_a (the set $f_a(\mathbf{p}_a)$ contains a finite number of points.

(b) Stress Constraints.

Next, certain structural limits must not be exceeded. For example, the maximum values of the stress resultants $\mathbf{f}$ and stress couples $\mathbf{m}$ must be kept below values which would cause yielding or other forms of damage. These result in inequalities of the form

$$\begin{gathered} \mathbf{m}_y(t,z_1) - \mathbf{m}_i(t,z_1,\mathbf{u}_1,\mathbf{u}_2,\mathbf{p}_s,\mathbf{p}_a) \le 0, \\ \forall\, z_1 \in [0,l]; \; \forall\, t \in [t^o{}_i, t^f{}_i]; \; i = 1,2,....,k , \end{gathered} \tag{2.27a}$$

and

$$\begin{gathered} \mathbf{f}_y(t,z_1) - \mathbf{f}_i(t,z_1,\mathbf{u}_1,\mathbf{u}_2,\mathbf{p}_s,\mathbf{p}_a) \le 0, \\ \forall\, z_1 \in [0,l]; \; \forall\, t \in [t^o{}_i, t^f{}_i]; \; i = 1,2,....,k , \end{gathered} \tag{2.27b}$$

where $\mathbf{m}_y$ and $\mathbf{f}_y$ represent "yield" or "failure" values and $\mathbf{m}_i$ and $\mathbf{y}_i$ are vectors of stress couples and stress resultants, respectively, during the $i-th$ manoeuver. Both of these depend on the state vector $\mathbf{x}$ and hence they

depend on both the control inputs and the structural and actuator parameters.

In addition, it may be necessary to limit relative values of the motion to prevent damage to on-board equipment, sensors, or measuring instruments. Relative displacements may be defined as the difference between $\mathbf{y}_i(t, z_1)$ and the motion of some "reference" location $\mathbf{y}_i(t, z_{1,ref})$ (e.g. $z_{1,ref}$ may be located at the center of mass of the undeformed beam). Accordingly, we define

$$\Delta \mathbf{y}_i(t, z_1, \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}_s, \mathbf{p}_a) \triangleq \mathbf{y}_i(t, z_1, \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}_s, \mathbf{p}_a) - \mathbf{y}_i(t, z_{1,ref}) \quad (2.28a)$$

and the relative translational motions by

$$\begin{aligned}\Delta \mathbf{w}_i(t, z_1, \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}_s, \mathbf{p}_a) &= \Delta \mathbf{y}_i(t, z_1, \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}_s, \mathbf{p}_a) \\ &- \langle \Delta \mathbf{y}_i(t, z_1, \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}_s, \mathbf{p}_a), \tau_1(t, z_{1,ref}) \rangle \tau_1(t, z_{1,ref})\end{aligned} \quad (2.28b)$$

and impose the constraint that

$$\begin{aligned}-\underline{w}(t, z_1)^2 + \langle \Delta \mathbf{w}_i(t, z_1, \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}_s, \mathbf{p}_a), \Delta \mathbf{w}_i(t, z_1, \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}_s, \mathbf{p}_a) \rangle \le 0, \\ \forall z_1 \in [0, l];\ \forall t \in [t^o{}_i, t^f{}_i];\ i = 1, 2, \ldots, k,\end{aligned} \quad (2.28c)$$

where $\underline{w}$ is a limiting motion.

(c) Constraints on Vibrations.

Finally, the controls $\mathbf{u}_1, \mathbf{u}_2$ must be such as to ensure that the motions "quiet down" as a manoeuver nears completion. The structural and actuator parameters must facilitate this task. Hence we get constraints of the form

$$\zeta_i(\mathbf{x}(t, z_1, \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}_s, \mathbf{p}_a)) \le 0,\ \forall z_1 \in [0, l],\ \forall t \in [t^o{}_i, t^f{}_i],\ i = 1, 2, \ldots, k. \quad (2.29)$$

(v) Formulation of Design Constraints: Small Displacements.

For small motion control, we propose to build a two-degrees of freedom control system, as in Fig. 6, with finite dimensional compensators whose free elements form an additional design vector $\mathbf{p}_c$. There seem to be three

main considerations in small motion behavior: stability, disturbance suppression and accuracy. In addition, speed of response may be of importance. In the case of a *finite dimensional system*, (see [Pol.11], attached), the requirement of stability can be expressed either as a semi-infinite inequality, by means of the Extended Nyquist Stability Criterion [Pol.6] or by imposing constraints on the eigenvalues of A. Disturbance rejection is ensured by imposing semi-infinite constraints on the disturbance to output transfer function of the closed loop system, $H_{yd}(j\omega)$ over a critical range of frequencies, and so forth.

In the case of a control system with a distributed plant, such as (2.24a), (2.24b), the computational aspects of the above techniques are still to be worked out and will form part of the proposed research.

(v) Complete Integrated LSS Optimal Design Problem.

Various cost functions can be considered in the design of an LSS, with the simplest one corresponding to minimization of the maximum energy used in a set of k large manoeuvers. A substantially more sophisticated cost function can be used to maximize stability robustness with respect to modeling errors, etc. In order to appreciate the full complexity of the optimal design problem, it is instructive to state the model problem corresponding to the energy minimization cost:

minimize

$$\max_{i \in \{1,2,\ldots,k\}} \int_{t^0_k}^{t^f_k} \sum_{z_1 \in f_a(\mathbf{p}_a)} (\mathbf{u}_{1,i}(t,z_1)^2 + \mathbf{u}_{2,i}(t,z_1)^2)\,dt \tag{2.30a}$$

subject to

$$\mathbf{L}^e \mathbf{x}(t^e{}_i, z_1, \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}_s, \mathbf{p}_a) = \mathbf{b}^e{}_i \quad \text{for } i = 1,2,\ldots,k\ ; \tag{2.30b}$$

$$L^f x(t^f_i, z_1, u_1, u_2, p_s, p_a) = b^f_i \text{ for } i = 1,2,\ldots,k; \tag{2.30c}$$

$$\underline{p}_s \le p_s \le \bar{p}_s; \tag{2.30d}$$

$$\underline{p}_a \le p_a \le \bar{p}_a; \tag{2.30e}$$

$$\|u_{j,i}(\cdot, z_1)\|_\infty \le \bar{u}_j(z_1), \ \forall z_1 \in f_a(p_a) \subset [0,l], \ j = 1,2; \text{ and } i = 1,2,\ldots,k \tag{2.30f}$$

$$\begin{aligned} &m_y(t, z_1) - m_i(t, z_1, u_1, u_2, p_s, p_a) \le 0, \\ &\forall z_1 \in [0,l]; \ \forall t \in [t^o_i, t^f_i]; \ i = 1,2,\ldots,k; \end{aligned} \tag{2.30g}$$

$$\begin{aligned} &f_y(t, z_1) - f_i(t, z_1, u_1, u_2, p_s, p_a) \le 0, \\ &\forall z_1 \in [0,l]; \ \forall t \in [t^o_i, t^f_i]; \ i = 1,2,\ldots,k; \end{aligned} \tag{2.30h}$$

$$\begin{aligned} &-\underline{w}(t, z_1)^2 + \langle \Delta w_i(t, z_1, u_1, u_2, p_s, p_a), \Delta w_i(t, z_1, u_1, u_2, p_s, p_a) \rangle \le 0, \\ &\forall z_1 \in [0,l], \ \forall t \in [t^o_i, t^f_i], \ i = 1,2,\ldots,k; \end{aligned} \tag{2.30i}$$

$$\zeta_i(x(t, z_1, u_1, u_2, p_s, p_a)) \le 0, \ \forall z_1 \in f_a(p_a), \ \forall t \in [t^o_i, t^f_i], \ i = 1,2,\ldots,k \tag{2.30j}$$

$$g^j(p_c, p_a, p_s, j\omega) \le 0, \ \forall \omega \in [\omega', \omega''], \ j = 1,2,\ldots,m, \tag{2.30k}$$

where $f_a(p_a)$ is a finite set of points at which the actuators are located, and the last set of constraints is of a form which results from linear control system design (see [Pol.11], attached).

(vi) Simulation Considerations.

The solution of the set of partial differential equations introduced in (iii) and (iv) above, presents considerable difficulties, even in the absence of the control or design aspects. Accordingly, it is normally necessary to utilize numerical methods to perform the simulations. The finite element method may be conveniently used to develop an approximate solution to the differential equations where the accuracy of the approximation is controlled by the number of elements used in the analysis, (e.g., see [Zie.1]). The effect of using the finite element method is to transform the problem from an infinite dimensional one (i.e.,

$\mathbf{x}(z_1, t)$) to a finite dimensional problem (i.e., $\mathbf{x}((z_1)_i, t)$, where i denotes a nodal position in the finite element model).

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